An Extract of ſome Phyſico-Mathematical Diſcourſes contained in Mr. COTES's Hydroſtatical and Pneumatical Lectures: printed for the uſe of thoſe that go the Courſe of Experiments.

The exact eſtimate of all manner of Preſſures. The invention of the Center of Preſſure upon any propoſed Plane reduced to the Problem of finding the Center of percuſſion.

WE are now to determine the quantity of that Preſſure which any ſurface ſuſtains that is expos'd to the Gravitation of a Fluid, this muſt be done gradually beginning with thoſe Caſes which are moſt ſimple and eaſy, and afterwards proceeding to thoſe which are more complex and difficult. Let a Veſſel *ABCD* (Fig. 1.) be propos'd containing any Fluid, ſuppoſe Water, let *AB* be the upper ſurface of the Water, and *CD* the Bottom of the Veſſel; the preſſure upon any part of that Bottom ſuppoſe *GH* will be equivalent to the weight of a Column of Water *GHIK*, having the part *GH* for its baſe and *GI* or *HK* the depth under Water for its altitude. This ſeems to be ſelf evident and may beſt be prov'd by the abſurdity of any contrary ſuppoſition; for if it be ſaid that *GH* ſuſtains a greater weight than that of the Column *GHIK*, the exceſs muſt come from the adjoyning Columns *ACGI* and *KHDB*, now for the like reaſon it ought to be ſaid that *CG* ſuſtains a greater weight than that of the Column *ACGI*, and *HD* a greater than that of the Column *KHDB*; but if this were true, then would all the parts *CG*, *GH* and *HD* together, of that whole plane *CD* ſuſtain a greater weight than that of the Columns together, or the whole Water which is above it namely *ACDB*, which is abſurd. The like abſurdity will follow if it be ſaid that *GH* ſuſtains a leſſer preſſure than the weight of the Column *GHIK*; the weight of that Column then, which is perpendicularly incumbent upon it is exactly equivalent to the preſſure which it ſuſtains. This is the Quantity of preſſure upon the Plane *GI* in the caſe that has already been deſcrib'd. If the Figure of the Veſſel be any way alter'd, the preſſure will ſtill be the ſame if the perpendicular diſtance of the Plane *GH* from the upper ſurface of the Water contain'd in the Veſſel of what ever Figure it be, remain unalter'd. Thus (in Fig. 2. 3.) if *LNGHOM* be a Veſſel of any irregular Figure, *LM* be the upper ſurface of the Water, and the Perpendicular diſtance of *GH* below *LM* namely *GI* or *HK* be the ſame as before; the preſſure of the Veſſelled Water *LNGHOM* upon the Bottom *GH* will be equal to the ſame weight of the Column *GHIK* as before, though the Veſſell'd Water *LGHM* be much leſs than that Column as in (Fig. 2.) or much greater as in (Fig. 3.) The preſſure is not to be eſtimated by the quantity of Water but by its Altitude. For if the quantity of Water *LGHM* (Fig. 2.) be a thouſand times leſs than *IGHK* as we may eaſily ſuppoſe it to be, and the quantity of Water *LGHM* (Fig, 3.) be a thouſand times greater than the ſame *IGHK*, then the quantity of this latter will be a million of times greater than the former, nevertheleſs both will equally preſs upon their bottoms *GH*, namely with a force equivalent to the weight of the perpendicular Column *GHIK*, which may deſervedly be accounted a paradox in Hydroſtaticks; but may thus (among other ways) be render'd intelligible. Let us conceive each of thoſe Veſſels plac'd in a larger *ABCD*, the preſſure upon *GH* will be the ſame whether we ſuppoſe the Water *LNGHOM* to be contain'd in its proper Veſſel *LNGHOM* or imagining that Veſſel to be away, we ſuppoſe its place to be ſupplyed by the ambient Water *ACGNL* and *HOMBD*: for any parcel of Water may be conceiv'd to be kept in by the reſt of the Water, which every way ſurrounds it as in a Veſſel, ſuppoſing all at reſt; now in this latter Caſe where we make the ambient Water *ACGNL* and *HOMBD* to be a Veſſel to the Water *LNGHOM* the preſſure upon *GH* is equivalent to the weight of the Column *GHIK* as has been already made out, therefore in the former where the Water *LNGHOM* was contain'd in its proper Veſſel, the preſſure upon *GH* will be alſo equivalent to the weight of the ſame Column *GHIK*. By the ſame way of reaſoning we may conclude that the Water contain'd in any other more irregular Veſſel as *LNGHOM* (Fig. 4.) preſſes upon the bottom with a force equivalent to the weight of the Column of Water *GHIK*, having the ſaid bottom for its Baſis, & *GI* or *HK* the perpendicular diſtance of the Plains *GH* and *LM* for its Altitude. If the Plane *GH* be oblique to the Horizon as in (Fig. 5.) The preſſure upon *GH* from the Water of the Veſſel *LNGHOM*, or from that of the Veſſel *EGHF*, or from that of the larger Veſſel *ACDB* will ſtill be the ſame, if the upper Surfaces *LM*, *EF* and *AB* be in the ſame Plane or at the ſame Altitude above *GH*. The Altitude is every where the meaſure of Preſſure whatever be the quantity of the Fluid, or however the containing Veſſel be figur'd.

I ſhould now proceed to eſtimate the preſſure upon Planes which are either perpendicular or oblique to the Horizon, but becauſe the ſeveral indefinitely ſmall Parts of which ſuch Planes are compoſed are acted upon with different forces accordingly as the particles of Water, by which they are immediately touch'd, happen to be at different depths, and ſince the total preſſure is made up of all theſe different forces taken together, we ought before we go any further to conſider what will be the preſſure which each of theſe indefinitely ſmall Parts ſuſtains. Firſt then we are to conſider that every ſmall particle of Water which is at reſt is preſs'd upon equally on all ſides by the other particles which ſurround it, otherwiſe it would yield to the ſtronger force till it were equally preſs'd every where, and as it is equally preſs'd on all ſides ſo does it every way by reaction equally preſs what ever is contiguous to it according to all poſſible contrary directions, for ſhould it preſs leſs than it were preſs'd it muſt neceſſarily yield to the force which is ſuppos'd greater then its own, and ſhould it preſs more than it were preſs'd, its force would neceſſarily remove its weaker Antagoniſt. Therefore ſince all things are ſuppos'd to be at reſt, we cannot any ways imagin this inequality of preſſure to take place. Now it has been prov'd before that the preſſure from above is equivalent to the weight of the incumbent Column of Water, therefore the preſſure from any other part or according to any other direction is alſo equal to the weight of the ſame incumbent Column, and ſince action and reaction are equal, the particle it ſelf muſt preſs according to all manner of directions, with the ſame force which is equivalent to the weight of the incumbent Column. 'Tis evident then that Fluids as they preſs according to all poſſible directions, ſo are the preſſures equal according to all directions, if the Point of contact in which the preſſure is made be at equal depths. This being allow'd we may proceed to what remains. Suppoſing then that *ACDB* (Fig. *6.*) is a Cubical Veſſel in which the Water reaches to the top, ſo that its upper Surface be repreſented by *AB*, let it be requir'd to determin the preſſure which one of its ſides *AC* does ſuſtain from the included Water. This ſide *AC* though repreſented here by a Line, to avoid confuſion in the Scheme, is ſuppos'd to be a Square. The meaſure of the preſſure upon every Point of that Square (or as it is here repreſented of that Line *AC*) is the Altitude of the Water above that Point; thus the preſſure upon *L* is meaſur'd by *AL*, the preſſure upon *M* by *AM*, the preſſure upon *N* by *AN*, and the preſſure upon *C* by *AC*, and the ſame may be ſaid for any other Points of the Line *AC*, therefore the preſſure upon the whole Line or upon all the Points of it will be meaſur'd by the Sum of ſo many of thoſe Altitudes *AL*, *AM*, *AN*, *AC*, as there are Points in the Line *AC*. Now that Sum may be thus eſtimated by drawing the Perpendicular *LO* equal to *AL* from the Point *L*, the Perpendicular *MP* equal to *MA* from the Point *M*, the Perpendicular *NQ* equal to *NA* from the Point *N*, and the Perpendicular *CD* equal to *CA* from the Point *C*. Now 'tis evident the Sum of *AL*, *AM*, *AN*, *AC* muſt be equal to the Sum of *LO*, *MP*, *NQ*, *CD*, and if from every intermediate Point between *A* and *L*, *L* and *M*, *M* and *N*, *N* and *C* Perpendiculars be conceiv'd to be drawn after the ſame method, the Sum of all thoſe Perpendiculars will be the meaſure of the total preſſure upon the Line *AC*. But the Sum of all thoſe Perpendiculars is equal to the Area of the Triangle *ACD*, therefore the Area of the Triangle *ACD* is the meaſure of the preſſure upon the Line *AC*. Now as the Line *AC* repreſents a Square ſo

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will the Triangle *ACD* repreſent a Priſm having the ſaid Triangle for its Baſe and the ſide of the Square for its Altitude. The weight of that Priſm of Water is therefore equivalent to the preſſure made againſt the Square or ſide of the Cube. That Priſm is equal to half the whole Cube as we learn from Euclid's Elements, therefore the preſſure againſt the Square is equivalent to half the weight of the whole Water contain'd in the Veſſel. There are 4 ſuch ſides of a Cube beſides the Top and Bottom, and each of thoſe 4 ſides for the ſame reaſon doth ſuſtain the ſame preſſure, therefore altogether do ſuſtain 4 times half the weight, that is twice the whole weight of the Water. And the Bottom by what has been prov'd above does it ſelf ſuſtain a preſſure equal to the whole weight of the Water; therefore the Bottom and Sides together of a Cubical Veſſel fill'd with Water do ſuſtain a preſſure from the Water equal to thrice the weight of it. I have endeavour'd to make the thing as eaſy as I believe the nature of it will permit. However ſince that part of this deduction where I told You the Triangle *ACD* did at the ſame time repreſent the Priſm when the Line *AC* repreſented the Square, might be perhaps a little obſcure I will endeavour to clear up this matter ſomething further. Let then *ACFE* (Fig. 7.) repreſent the Square Side of the Veſſel, and *CDGF* repreſent the Square Bottom of the ſame. It was prov'd before that the preſſure exercis'd upon the Line *AC* was meaſur'd by the Triangle *ACD*, by the ſame way of reaſoning it may be prov'd that the preſſure upon the Line *EF* is meaſur'd by the Triangle *EFG*, and the preſſure upon any other Line *HI* which is parallel to theſe two and ſituated between them is meaſur'd by its reſpective Triangle *HIK*. If we imagin the Square *ACFE* to be made up of an Infinite Number of ſuch intermediate Lines as *HI*, the preſſure upon the whole Square will be made up of the ſame Infinite Number of ſuch equal Triangles as *HIK*, now the Sum of all thoſe Triangles make up the Priſm *AEGDCF* and this Priſm is half the whole Cube, as in the former Scheme the Triangle *ACD* is half the Square *ACDB*. If the Plane *ACFE* inſtead of being a Square were a rectangled Parallelogram having its Sides *AE* either longer or ſhorter than *AC*, it would follow from the Principles that the preſſure to which it is expos'd would be equivalent to the weight of a like Priſm of Water, having the Triangle *ACD* for its Baſis, and the Side *AE* for its Altitude.

I have been hitherto ſpeaking of Planes which are either parallel or perpendicular to the Horizon, it will be no difficult matter to apply what has been ſaid of Perpendicular Planes to thoſe which are oblique. Let *AC* (Fig. 8.) repreſent any ſuch oblique Plane and let the upper Surface of the Water be *AB*. The meaſure of the preſſure upon the Point *L* is *LS* the Altitude of the Water above that Point, ſo *TM* is the meaſure of the preſſure upon *M*, *VN* the meaſure of the preſſure upon *N*, and *XC* the meaſure of the preſſure upon *C*. Erect the Perpendiculars *LO*, *MP*, *NQ*, *CR* equal reſpectively to *LS*, *MT*, *NV*, *CX*, and imagin the like Conſtruction to be made for all the other Points of the Line *AC*, and the ſum of all thoſe Perpendiculars, that is the Triangle *ACR* will be the meaſure of the preſſure upon the whole Line *AC*. If this Line *AC* be ſuppos'd to repreſent a Parallelogram as before, then the Triangle *ACR* will as before become a Priſm, and the weight of that Priſm of Water, which we are taught by Euclid how to meaſure, will be the preſſure ſuſtain'd by the Parallelogram.

I have hitherto ſuppos'd that the Line *CA* or the Parallelogram repreſented by it coincides with the Surface of the Water at *A*; if that does not happen but the higheſt part of the Line or Parallelogram is at ſome diſtance from the Surface, a Computation of the preſſure will ſtill be eaſy enough. Suppoſe *MC* in the 8 Figure were the Line or Parallelogram propos'd, the Preſſure upon the Line *MC* will be meaſur'd by the Trapezium or four ſided Figure *MCRP*, and the preſſure upon the Parallelogram repreſented by that Line will be a Priſm, having that Trapezium for its baſe, and the other ſide of the Parallelogram which is ſuppos'd parallel to the Surface of the Water for its Altitude.

From what has been ſaid of theſe few particular Inſtances we may now underſtand, that the preſſure upon any Plane of what ever Figure and Situation is equivalent to the weight of a Solid of Water, which is form'd by erecting Perpendiculars upon every point of the Plane propos'd, equal to the reſpective diſtances of thoſe points from the upper Surface of the Water. For the Perpendiculars being the meaſure of the preſſure upon the Points from which they are erected, the Sum of thoſe Perpendiculars, or the Solid form'd by them, will be equal to the Sum of the preſſures upon the Points, or the total preſſure upon the whole Plane.

Or we may thus expreſs the ſame thing after another way, and ſo take in all curv'd Surfaces as well as Planes, that the preſſure upon any Surface is equal to the Sum of all the Products which are made by multiplying every indefinitely ſmall part of the Surface into its diſtance from the top of the Water. For the preſſure upon each of thoſe Parts is equal to a Column of Water having the Part for its Baſis and the diſtance from the top of the Water for its Altitude, and every one knows who has the leaſt skill in Geometry, that thoſe Columns are meaſur'd by multiplying their Baſes by their Altitudes, therefore the Sum of the Products of all thoſe Baſes or little Parts by their Altitudes or reſpective diſtances from the top of the Water will be equal to all the Columns upon every little Part, and therefore to a Body of Water whoſe weight will be equivalent to the total Preſſure upon the whole Surface. Now to find the Sum of all theſe Products or a Body of Water equal to that Sum, is a very difficult Problem in moſt Caſes. *Stevinus* in his Hydroſtaticks has attempted it only in a few Inſtances, and thoſe of plane Surfaces, and amongſt plane Surfaces he meddles only with ſuch which he calls regular, nevertheleſs he has gone the furtheſt in this matter of any Writer I have met with. To ſupply then this defect I will here lay down another Rule, which is not only univerſal, but alſo as eaſy and expeditious as can be deſir'd. It is this, that the Preſſure upon any Surface whatever; however it be ſituated is equal to the weight of a Body of Water whoſe magnitude is ſound by multiplying the Surface propos'd into the depth of its Centre of Gravity under Water. So the preſſure upon any number of Surfaces of different Bodies, however differently ſituated, is equal to the weight of a Body of Water whoſe magnitude is found by multiplying the Sum of all thoſe Surfaces into the depth of their common Centre of Gravity under Water. The Demonſtration of this Rule may not perhaps be fully underſtood by thoſe who are unacquainted with Staticks and the Nature of the Centre of Gravity, however I will here produce it, that thoſe who can may underſtand it, and that others taking now for true what I ſhall aſſume as demonſtrated by the Writers of Mechanicks may afterwards be fully ſatisfied of it, when they come to underſtand that Theorem it is grounded upon; which is, that if every indefinitely ſmall Part of any Surface or number of Surfaces be multiplyed reſpectively into its Perpendicular diſtance from any propos'd Plane, the Sum of thoſe Products will be equal to the Product of the whole Surface or number of Surfaces multiply'd into the Perpendicular diſtance of the Centre of Gravity of the ſingle Surface, or of the common Centre of Gravity of the whole number of Surfaces from the ſame Plane. Now taking the upper Surface of Water for that Plane to which we refer the indefinitely ſmall Parts of the Surface which is expos'd to the preſſure we are concern'd with, ſince it has been already ſhewn that the preſſure upon the whole is equivalent to the weight of a Body of Water which is equal in magnitude to the Sum of all the Products, made by multiplying every little Part by its diſtance from the upper Plane of the Water, and this Sum of Products is by the Statical Theorem I have been mentioning exactly equal to the Product of the whole Surface or number of Surfaces multiplyed into the diſtance of the Centre of Gravity from the upper Plane of the Water, it will follow that the ſame Product is the meaſure of a magnitude of Water whoſe weight is equivalent to the preſſure requir'd. The ſame Rule may be demonſtrated by ſeveral other methods, but I have pitch'd upon this as the fitteſt for my purpoſe.

Another thing which *Stevinus* propoſes to himſelf is to determin the Centre of preſſure upon any Plane. Before we can diſcourſe any further about this we muſt declare what is meant by that Centre. It is then the point to which if the total preſſure were apply'd, its effect upon the Plane would be the ſame as when it was diſtributed unequally over the whole after the manner before deſcrib'd; or we may ſay it is that Point in which the whole preſſure may be conceiv'd to be united; or it is that point to which if a force were applyed, equal to the total preſſure but with a contrary direction, it would exactly ballance or reſtrain the Effect of the preſſure. Thus if *ABCD* (Fig. *9.)* as before be a Veſſel of Water, and the ſide *AC* be preſs'd upon with a force equivalent to twenty pounds of Water; this force we have ſeen is unequally diſtributed over *AC*, for the parts near *A* being at a leſſer depth are leſs preſs'd upon than the parts near *C* which are at a greater depth, and therefore the efforts of all the particular preſſures are united in ſome point *Z*, which is nearer to *C* than to *A*, and that point *Z* is what may be call'd the Centre of preſſure: if to that point a force equivalent to twenty pound weight be apply'd it will affect the Plane *AC* in the ſame manner that it was affected before by the preſſure of the Water diſtributed unequally over the whole. And if to the ſame point we apply the ſame force with a contrary direction to that of the preſſure of the Water, the force and preſſure will ballance each other, and by contrary endeavours deſtroy each others effects. Suppoſe at *Z* a Cord *ZPW* were fix'd, which paſſing over the Pulley *P*, has a weight *W* of twenty pounds annex'd to it, and that the part of the Cord *ZP* were perpendicular to *AC*, the effort of the weight *W* is equal and its direction contrary to that of the preſſure of the Water. Now if *Z* be the Centre of preſſure theſe two Powers will be in Equilibrio, & mutually defeat each others endeavours. It may be worth while to be acquainted with a Rule for finding that Centre in all Caſes. We cannot have much help from *Stevinus* in this buſineſs, he undertakes only a few particulars and thoſe which are the eaſieſt, ſuppoſing that his Reader will apply the like method to other Circumſtances, but they who ſhall endeavor to make ſuch an Application will in moſt Caſes find it more difficult than they might poſſibly expect. I have for that reaſon devis'd this general Rule which follows. That if any Plane which happens to be propos'd be produc'd till it interſect the upper Surface of the Water produc'd, if need be, and the Line which is the common Section of the two Planes be made an Axis of Suſpenſion, the Centre of Oſcillation or Percuſſion of the Plane as it is ſuppos'd to revolve about that Axis will be the Centre of preſſure requir'd. Thus if *AC* (Fig. 10.) repreſents the Plane propos'd let it be produc'd till it cut the Plane *HG* in *D*, now if *D* be made the Axis of Suſpenſion of the Plane *AC*, the Centre of percuſſion of the Plane *AC* revolving about *D* will be alſo the Centre of preſſure upon the ſame Plane. For if the percuſſive forces of every point of *AC* be as the preſſures exercis'd upon thoſe points, then the Centre of percuſſion muſt needs be the ſame with the Centre of preſſure, and that the force of percuſſion is every where as the preſſure of the Water may thus be prov'd. The percuſſive force of any point ſuppoſe *B* is as the velocity of that point, and the velocity is as the diſtance *BD* of the point from the Axis of Motion; ſo the percuſſive force of *A* is as *AD*, of *C* as *CD*, ſince then the percuſſive forces of *A*, *B*, *C* are as the Lines *DA*, *DB*, *DC* and thoſe Lines are as the Lines *EA*, *FB*, *GC* perpendicular to the Surface of the Water, and theſe laſt Lines are as the preſſures upon *A*, *B*, and *C*, it follows that the percuſſive forces taking the interſection *D* for the Axis of ſuſpenſion or motion are reſpectively as the preſſures upon the ſame points: therefore the Centre of Percuſſion or Oſcillation is the ſame with the Centre of preſſure. The Geometers of the laſt Age have proſecuted the Problem of finding the Centre of Oſcillation very diligently, being excited thereto chiefly by the noble Invention of Pendulum Clocks: the Rules they have laid down for that purpoſe are eaſy enough, and the Applications they have actually made of thoſe Rules are not a few. Having therefore ſhewn how the Centre of Oſcillation may be made uſe of for determining the Centre of preſſure, I preſume I have by this time ſufficiently clear'd up what I propos'd. For further illuſtration I will add a couple of Examples.

Let it be requir'd to find the preſſure which a Diver ſuſtains when the Centre of Gravity of the Surface of his Body is 32 feet under Water. The Surface of a middle ſiz'd humane Body is about 10 ſquare feet. Multiply then 32 the depth of the Centre under Water by 10 the Surface of the Body, and the product or 32 times 10 ſolid feet will be a magnitude of Water whoſe weight is equivalent to the preſſure which the Diver ſuſtains by the Rule before laid down. A Cubick foot of Water has been found by Experiment to weigh 1000 Averdupois Ounces, therefore 32 times 10 feet or 16 times 20 feet of Water will weigh 16 times 20000 Averdupois Ounces or 20000 Averdupois Pounds. This therefore is the preſſure of the Water to which a Diver at 32 feet depth is expos'd. Again in (Fig. 11.) let the right angled Parallelogram *ABCD* be a Wall, Dam, or Pen of Timber perpendicular to the Horizon made to keep in a Pond of Water, whoſe upper Surface reaches to *AB*: let *AB* be 20 feet, and *AC* 12. Let *K* be the Centre of Gravity of the Plane, the depth of that Centre *K* will be equal to half *GH* or half *AC*, that is 6 feet. The Area of the Plane is found by multiplying *AC* by *AB* or 12 by 20, it is therefore 240 ſquare feet: multiply according to the Rule the Area 240 by *GK* which is 6 and the product will be 1440 Cubick feet of Water, which weighs ſo many thouſand Ounces that is 90000 pounds, and that is the preſſure which the Dam *ABCD* ſuſtains. To find the Centre of that preſſure we muſt make the Line *AB* which is the common Section of the Dam and the upper Surface of the Water, the Axis of ſuſpenſion of the Plane *ABCD*; now it appears by the diſcovery of *Hugens*, *Wallis* and other Geometers that *Z* the Centre of Oſcillation of this Plane ſo ſuſpended will be in the Line

GH which biſects this Plane and is parallel to *AC* or *BD*, and the Line *GZ* will be two thirds of *GH* or *AC* that is 8 feet and the ſame point *Z* ſo determin'd is, as was prov'd before, the Centre of preſſure requir'd.

An Enquiry into the Limits and State of the Atmoſphere.

HAVING Yeſterday made it appear from reaſon that the Spring or Elaſtick Power of the Air is as the force which compreſſes it, and having this Day as far as the unavoidable irregularity of Tubes would permit us, ſhewn by ſeveral Experiments that the Denſity is alſo as the ſame force, the Space it poſſeſſes being always reciprocally as that force; We are now furniſh'd with ſufficient *data* to make our Enquiries concerning the Limits of the Atmoſphere, and to determine its State as to Rarity at different Elevations from the Earth's Surface. If the Air were of the ſame conſiſtence as to its Rarity or Denſity at all Altitudes it would be no difficult thing to ſet bounds to it. We collected from the Experiment which was Yeſterday made at the top and bottom of the Obſervatory, that the Specifick Gravity of Water is about 850 times greater than the Specifick Gravity of Air, (which thing will hereafter be further examin'd by an Experiment particularly fitted for that purpoſe) and in the foregoing Week we found by the Hydroſtatical Balance, that Quickſilver is about 14 times heavier than Water; it follows then of conſequence that Quickſilver is 14 times 850 degrees heavier than Air, that is, 11900 times heavier. We have ſeen by the Torricellian Experiment that a Column of Quickſilver of 29½ Inches is uſually a counterpoiſe to a Column of Air having the ſame baſis and reaching to the top of the Atmoſphere; if therefore the Air be every where of the ſame Denſity as it is here below, its Altitude ought as many times to exceed the height of 29½ Inches (which is the height of an Æquiponderant Column of Quickſilver) as its Specifick Gravity falls ſhort of the Specifick Gravity of Quickſilver; that is, the Atmoſphere ought (upon the ſuppoſition of an every where uniform Denſity) to be 11900 times 29½ Inches or ſomewhat above 5½ Miles high. But it may be eaſily prov'd that this ſuppoſition does in no wiſe take place. For ſince every Region of the Air is compreſt by that part of the Atmoſphere which is ſuperior to it, and ſince the higher Parts have a leſſer weight incumbent upon them than the lower, and ſince the Denſity of the Air is every where as the force which compreſſes it, it will follow of neceſſity that there is ſtill a greater Rarity of the Air as it is further diſtant from the Surface of the Earth. How far the Air may poſſibly admit of Rarefaction and Condenſation has not yet that I know of been determin'd by any one. Mr. *Boyl* has obſerv'd that it may be ſo dilated as to become 10000 times rarer than it is in its natural State.

Dr. *Halley* ſays that he himſelf has ſeen the Air compreſs'd ſo as to be 60 times denſer than it is as we commonly breath it; and Monſieur *Papin* relates that he was a Witneſs that Monſieur *Hugens* did once in a Glaſs Veſſel compreſs the Air to the ſame degree before the Glaſs was broken, yet never could any Experimenter determin how much further the Air might poſſibly be rarefied or condens'd. However it's certain that there are in Nature ſome Limits which cannot be exceeded. No condenſation can reach ſo far as to cauſe a penetration of Parts, and if the Rarefaction of the Air be ſtill greater, as its diſtance from the Surface of the Earth increaſeth, its Spring will at length be ſo weakn'd that the force, with which every Particle of it endeavours to tend upwards from the Particles which are next below it, will be weaker than the force of its own gravity which endeavours conſtantly to detain it. The Rarefaction of the Air muſt therefore be bounded of neceſſity when theſe two oppoſite forces come to ballance each other. Though this be certainly true that the Air can't poſſibly expand it ſelf beyond a certain meaſure upon account of its Gravity, yet ſince Men have not hitherto been able to ſet any bounds to its utmoſt expanſion, it is equally certain, that we cannot poſſibly define the Limits of the Atmoſphere. For as the Air may be more and more rarefied, ſo will the ſame Quantity of it (which equals the weight of about 30 Inches of Quickſilver) be contain'd in a greater Space, and thereby thoſe Limits be ſo much the wider. Notwithſtanding this ſeeming difficulty we may ſtill collect how much the Air is rarified at any propos'd Altitude from the Surface of the Earth after the following manner. Let $XA\alpha PX$ (Fig. 14.) repreſent a Veſſel reaching from the Surface of the Earth $A\alpha$ to the top of the Atmoſphere X. Let us imagin the ſide AX divided into Inches AB, BC, CD &c. and let the Lines BK, CL, DM, EN &c be drawn parallel to $A\alpha$. 'Tis evident that the Air contain'd between BK and CL is rarer than the Air contain'd between $A\alpha$ and BK; the former having a leſſer Column of Air $XCLX$ incumbent upon it, than the Column $XBKX$ which preſſes upon the latter. Upon the ſame account the Air between CL and DM is rarer than that between BK and CL, and that between DM and EN rarer than that between CL and DM. And thus is every ſuperior Inch of Air rarer than that below it. Let us now ſuppoſe, that every Inch of Air is in all parts of it of an equal Denſity, or that the Air AK is every where uniform, but denſer than the Air BL; which is alſo ſuppos'd to be every where uniform, but denſer than CM; and that to be uniform it ſelf, but denſer than DN, and ſo onwards. Again let us ſuppoſe that the Air BL is reduc'd to a leſſer Space BQ, ſo as to become equally denſe with the Air AK, which is done by making the Space BQ leſſer than BL, in the ſame proportion that the Air BL is leſs denſe than the Air AK; after the ſame manner let the Air CM be reduc'd to the Space CR, and the Air

DN to the Space *DS* and ſo onwards, that thus every Inch of Air may be reduc'd to the ſame conſiſtence with the Air *AK*; now it is evident from this conſtruction that the Spaces *AK*, *BQ*, *CR*, *DS*, &c will every where be as the Denſities reſpectively of the ſeveral Inches of Air *AK*, *BL*, *CM*, *DN*; and it is alſo evident that the Quantity or weight of the Air which reaches from any one of thoſe Spaces up to the extremity of the Atſphere will every where be as the Sum of all the Spaces which are ſituated above the Space propos'd. Thus the Quantity or the weight of Air above the Space *AK* will be as the Sum of the Spaces *BQ*, *CR*, *DS*, *ET*, *FV* &c, and the Quantity or weight of Air above the Space *CR* will be as the Sum of the Spaces *DS*, *ET*, *FV* &c. For the Air being every where reduc'd to the ſame conſiſtence, the Quantity or weight of it will be as the Space it poſſeſſes. Theſe things being laid down I may now without much difficulty proceed to eſtabliſh the Concluſion I aim at, which is this, that if any number of diſtances from the Surface of the Earth be taken in an Arithmetical Progreſſion, the Denſities of the Air at thoſe Diſtances will be in a Geometrical Progreſſion. For ſince by the Experiments which have this Day been made, it appears that the Denſity of the Air is always as the force which compreſſes it, we muſt conclude that the Denſity of the Air at any Diſtance from the Surface of the Earth is as the Quantity or weight of that part of the Atmoſphere which is above it. Therefore in our Scheme the Denſities of the Air between *Aα* and *BK*, *BK* and *CL*, *CL* and *DM* &c. are to each other reſpectively as the Quantities of Air Above *Aα*, *BK*, *CL* &c. up to the extremity of the Atmoſphere. But we ſaw before that thoſe Denſities were as the Spaces *AK*, *BQ*, *CR* &c. reſpectively, and thoſe Quantities of Air reaching to the extremity of the Atmoſphere were as the Spaces *XBβQRSTVX*, *XCγRSTVX*, *XDδSTVX* reſpectively, it follows then that the Spaces *AK*, *BQ*, *CR* are to each other reſpectively as the Spaces *XBβQRSTVX*, *XCγRSTVX*, *XDδSTVX*. Now the former Spaces *AK*, *BQ*, *CR* are the differences of the latter, and it is well known to thoſe who underſtand any thing of the nature of Proportions, when any ſet of Quantities are to each other reſpectively as their differences that then as well the Quantities themſelves as their differences, are in a Geometrical Progreſſion. The Spaces *AK*, *BQ*, *CR*, are therefore in a Geometrical Progreſſion, as the Diſtances *AB*, *AC*, *AD* are in an Arithmetical Progreſſion. And as the Denſities of the Air belonging to theſe three firſt Inches are in a Geometrical Progreſſion, ſo do the Denſities of the Air belonging to every one of the other Inches, which are ſuppos'd to be continued up to the extremity of the Atmoſphere, decreaſe in the ſame Geometrical Progreſſion, as any one without difficulty may collect by the ſame way of reaſoning. I have hitherto ſuppoſed for eaſe of conception that the Air is of the ſame Denſity in every part of each Inch of Altitude, nevertheleſs it is certain that every the leaſt variation of Altitude cauſes a variation of Denſity in the Air. The Concluſion however will not hereby be diſturb'd: for if inſtead of dividing the Altitude of the Atmoſphere into Inches as before, we conceive it now to be divided into its moſt indefinitely minute parts, applying to theſe what we have ſaid above concerning the Inches, we ſhall at length deduce the ſame Geometrical Progreſſion of Denſities anſwering to a like Arithmetical Progreſſion of Altitudes. Now becauſe the Rarity of any Body is reciprocally as its Denſity, we may alſo conclude that as the Diſtances from the Surface of the Earth do increaſe in an Arithmetical Progreſſion, ſo do the different degrees of Rarity of the Air increaſe in a Geometrical Progreſſion. This Property of the Air was firſt that I know of obſerv'd by Dr. *Halley*, but becauſe his Demonſtration cannot be underſtood by thoſe who are unacquainted with the nature of the Hyperbolick Line, and Dr. *Gregory* in his Demonſtration of the ſame thing, which may be ſeen in the fifth Book of his Aſtronomy, ſuppoſes his Reader to be furniſh'd with ſo much Geometry as not to be ignorant of the properties of the Logarithmick Line, I have endeavour'd to make the thing intelligible by a Method which may be eaſy even to thoſe who have never medled with Curvilinear Figures. Let us ſee now what help we have from this property to determin how much the Air is really rarefied at any propos'd Elevation from the Surface of the Earth. Since the Elevations are the Terms of an Arithmetical Progreſſion as the Rarities are the Terms of a Geometrical, it follows, that the Elevation is every where proportionable to the Logarithm of the Rarity. If then by Experiment we can poſſibly find the Rarity of the Air at any one Elevation, we may by the Rule of Proportion find what is the Rarity at any other propos'd Elevation: by ſaying, as the Elevation at which the Experiment was made, is to the Elevation propos'd, ſo is the Logarithm of the Air's Rarity which was obſerv'd at the Elevation where the Experiment was made, to the Logarithm of the Air's Rarity at the Elevation propos'd. Thus I collected from the celebrated French Experiment of the *Puy de Domme*, which I Yeſterday gave You an Account of, that at the Altitude of 7 Miles the Air is ſomewhat above 4 times rarer than at the Surface of the Earth. By the ſame method I collected from the Experiment of Mr. *Caſwell* made upon *Snowdon Hill*, that at the ſame Altitude of 7 Miles the Air is not altogether ſo much as 4 times rarer than at the Surface, the difference on both ſides was inconſiderable. We may take a mean therefore and ſay in a round number, that at the Altitude of 7 Miles the Air is about 4 times rarer than at the Surface of the Earth. Sr. *Iſaac Newton* in his late Additions to his Opticks makes uſe of this very ſame proportion, what grounds he went upon is difficult to gueſs, however I am ſatisfied of the Concluſion from my own Computation. Now from what has been already prov'd that the

Rarity of the Air is augmented in a Geometrical as the Altitude is augmented in an Arithmetical Progreſſion, it follows that every ſeven Miles added to the Altitude does always require a rarity of the Air ſtill 4 times greater. Therefore at the Altitude of 14 Miles the Air is 16 times rarer than at the Surface, at the Altitude of 21 Miles it is 64 times rarer, at the Altitude of 28 Miles 256 times, at 35 Miles 1024 times, at 70 Miles about a Million of times, at 140 Miles a Million of Million of times, at 210 Miles a Million of Million of Millions of times, if the Air can poſſibly expand it ſelf to ſo large Dimenſions. Hence we may eaſily gather that the Air at the Altitude of 500 Miles (if the Atmoſphere can reach ſo far) muſt neceſſarily be there ſo much rarified that if a Globe of the Air we breath in of an Inch Diameter were as much dilated it would poſſeſs a larger Space than the whole Sphere of Saturn. The Semidiameter of the Earth is nearly 4000 Miles which is 8 times 5 hundred Miles: with good reaſon then might that excellent Philoſopher I have lately been mentioning tell us in his *Principia*, that the Air at the Altitude of a Semidiameter of the Earth is at leaſt ſo wonderfully rarified as I have deſcrib'd it to be at an Altitude 8 times leſs.

It appears from the Obſervations of Aſtronomers of the Duration of Twilight and of the magnitude of the Terreſtial Shadow in Lunar Eclipſes, that the Effect of the Atmoſphere to reflect and intercept the light of the Sun is ſenſible even to the Altitude of between 40 and 50 Miles, ſo far then may we be certain that the Atmoſphere reaches, and at that Altitude we may collect from what has been already ſaid that the Air is about 10000 times rarer than at the Surface of the Earth. How much farther than this Altitude of between 40 and 50 Miles the Atmoſphere may be extended I muſt confeſs I am altogether ignorant, there bring no *data* that I know of from which a greater Altitude may be indubitably concluded. There has indeed been often ſeen in the Atmoſphere ſome very Luminous parts even near the Zenith about Midnight, but I dare not conclude any thing from ſuch appearances; if I ſhould aſſert as ſome have done that theſe luminous parts are nothing elſe but ſome Terreſtial Exhalations floating in the Air at a prodigious Altitude, and thereby reflecting the Light of the Sun which they are expos'd to at that great height to our eyes, it will be next to impoſſible to give any tolerable account how thoſe Exhalations can be denſe enough to reflect ſo copious a Light at that vaſt diſtance, and at the ſame time be ſupported by a Medium, I may ſay, almoſt infinitely rarer than the Air we breath in. It ſeems more probable that theſe extraordinary Lights proceed from ſome ſelf ſhining Subſtance or Aerial *Phoſphorus*. A ſurprizing appearance of this kind was ſeen at *Cambridge* about 10 of the Clock at Night and at other very diſtant Places on the 20*th* of *March* in the Year 1706. It was a Semicircle of Light of about two thirds of the ordinary breadth of the milky way but much brighter. The top of it paſs'd very near our Zenith inclining about 4 or 5 Degrees to the North, it croſs'd the Horizon at a very ſmall diſtance from the Weſt towards the South, and again about as far from the Eaſt towards the North. It was moſt vivid and beſt defin'd about the Weſtern Horizon and moſt faint about the Zenith, where it firſt began to diſappear : there was at the ſame time an *aurora Borealis.* A Friend of mine ſaw the ſame appearance in *Lincolnſhire* at the diſtance of about 70 Miles north of *Cambridge*: the Semicircle ſeem'd to Him to lye in the Plane of the Æquator. From theſe two obſervations compar'd together it is eaſy to collect that the matter from which that Light proceeded was elevated above the Earth's Surface between 40 and 50 Miles. Having now finiſh'd what I deſign'd to repreſent concerning the Limits and different degrees of rarity of the Atmoſphere at different Altitudes I might here conclude. But becauſe it may poſſibly be expected that I ſhould add ſomething in this place concerning the Cauſe of the Airs Elaſticity upon which theſe deductions were grounded, it may not be amiſs to declare here that of all the ſeveral Hypotheſes which I have hitherto been ſuggeſting for this purpoſe, that of Sir *Iſaac Newton* ſeems to me to be the moſt probable. He has demonſtrated in the ſecond Book of his *Principia* that if the particles of the Air be of ſuch a Nature as to recede from each other with centrifugal forces reciprocally proportionable to their diſtances, they will compoſe an Elaſtical Fluid whoſe Denſity will always be as the force which compreſſes it; and any one who reads the late Additions to his Opticks will perceive that that Hypotheſis is not advanc'd without reaſon.

An account of the ſeveral ſucceſſive Degrees in which the Air is expanded and compreſſed by the Air-pump and Condenſer. Wherein the firſt and ſecond Tables are explained.

AT our laſt meeting we took a particular view of the ſeveral parts of which our Engines conſiſt. I ſhall therefore ſuppoſe You to be ſufficiently acquainted with the Fabrick and contrivance of them, and to underſtand in general the manner of their Operations. I ſay in general, becauſe there are ſome particulars which yet remain at this time to be diſcours'd of, which may alſo very well deſerve your conſideration and will be of good Uſe in order to frame juſt and true apprehenſions of the Experiments which will hereafter be made. I ſhall begin with the Air-pump, and repreſent to You by what degrees the Air contain'd in the Receiver is exhauſted.

It may perhaps upon the firſt view ſeem not improbable that an equal evacuation is made at each ſtroke of the Pump and conſequently that the

Receiver may after a certain number of Strokes be perfectly exhauſted; for it muſt be allow'd, if an equal quantity of Air is taken away at every ſtroke, that the Receiver will in time be perfectly exhauſted, how ſmall ſoever thoſe equal quantities which are continually taken away, may be ſuppoſed to be. Thus if the Air which goes out of the Receiver at each turn of the Pump be but the Hundredth part of what was at firſt included in the Receiver; 'tis certain that a total evacuation will be made after an Hundred turns. That things are thus may at firſt view I ſay ſeem not improbable. But if we conſider the matter more nearly we ſhall find it to be far otherwiſe.

What I ſhall endeavour to make out to you is this; that the quantities exhauſted at every ſtroke are not equal but are perpetually diminiſh'd and grow leſſer always ſo long as you continue to work the Pump: that no Receiver can ever be perfectly and intirely evacuated, how long time ſoever you imploy for that purpoſe, notwithſtanding that the Engin be abſolutely free from all defects and in the greateſt perfection which can be imagin'd. It may appear to be a Paradox, that a certain quantity of the Air in the Receiver ſhould be removed at every turn of the Pump and yet that the whole can never be taken away; but I hope I ſhall eaſily ſatisfy you that it is not a Miſtake. Laſtly, that I may not ſeem too much to depreciate the Value of our Engin, I have this further to ſay for it: that though it be impoſſible by it's means to procure a perfect Vacuum, yet you may approach as near to it as you pleaſe. By a perfect Vacuum here I mean in reſpect of Air only, not an abſolute Vacuity in reſpect of every thing which is Material: for not to mention what other ſubtile Bodies may poſſibly be lodg'd in our emptied Receivers, 'tis matter of fact that the Rays of Light are not excluded from thence.

In Order to make out theſe Aſſertions, I ſhall in the firſt place lay down this Rule. That the quantity of Air which is drawn from the Receiver at each ſtroke of the Pump, bears the ſame proportion to the quantity of Air in the Receiver immediately before that ſtroke, as the capacity of the Barrel into which the Air paſſes from the Receiver, does to the capacity of the ſame Barrel and the capacity of the Receiver taken together. You may remember that in each Barrel there are two Valves, whereof the lower is placed at the bottom of the Barrel and the upper is fix'd upon the Embolus or Sucker. Now the hollow Space which lies betwixt theſe Valves, when the Embolus is rais'd as high as it can go, is what I call the capacity of the Barrel: for the other part of the cavity of the Barrel, which is above the Embolus and the upper Valve, is of no uſe in evacuating the Receiver, and therefore ought not here to be conſidered. Upon a like account, by the capacity of the Receiver, I mean, not only the Space immediately contained under the Receiver, but alſo all thoſe other hollow Spaces which communicate with it, as far as to the lower Valves: ſuch you may remember are the cavity of the Pipe which conveys the Air to the Barrels, and the cavity in the upper part of the Gage above the Quickſilver. Theſe additional Spaces are very ſmall and inconſiderable; yet if we would be exact, they alſo muſt be taken into the account and look'd upon as parts of the Receiver. Now to underſtand the truth of this Rule, we muſt obſerve that as the Embolus is moved upwards from the bottom of the Barrel, it would leave a void Space behind it, but this effect is prevented by the ruſhing in of Air from the Receiver. The Air you know, by it's Elaſticity, is always endeavouring to expand it ſelf into larger Dimenſions, and it is by this endeavour that it opens the lower Valve and paſſes into the hollow part of the Barrel as the Embolus gives way to it, and this it will continue to do, till it comes to have the ſame Denſity in the Barrel as in the Receiver: for ſhould its Denſity in the Barrel be leſs than in the Receiver, its Elaſtick force which is proportionable to its Denſity would be leſs alſo, and therefore it muſt ſtill give way to the Air in the Receiver till at length the Denſities become the ſame. The Air then which immediately before this ſtroke of the Pump, (by which the Sucker is raiſed) was contained in the Receiver only, is now uniformly diffuſed into the Receiver and the Barrel; whence it appears that the quantity of Air in the Barrel is to the quantity of Air in the Barrel and Receiver together as the capacity of the Barrel is to the capacity of the Barrel and Receiver together. But the Air in the Barrel is that which is excluded from the Receiver by this ſtroke of the Pump, and the Air in the Barrel and Receiver together is what was in the Receiver immediately before the ſtroke. Therefore the truth of the Rule is very evident: that the quantity of Air which is drawn from the Receiver at each ſtroke of the Pump bears the ſame proportion to the quantity of Air in the Receiver immediately before that ſtroke, as the capacity of the Barrel into which the Air paſſes from the Receiver, does to the capacity of the ſame Barrel and the capacity of the Receiver taken together. To illuſtrate this further by an Example: let us ſuppoſe the capacity of the Receiver to be twice as great as the capacity of the Barrel; then will the capacity of the Barrel be to the capacity of the Barrel and Receiver together as 1 to 3, and the quantity of Air exhauſted at each turn of the Pump is to the quantity of Air which was in the Receiver immediately before that turn, in the ſame proportion. So that by the firſt ſtroke of the Pump, a third part of the Air in the Receiver is taken away; by the ſecond ſtroke a third part of the remaining Air is taken away, by the third ſtroke a third part of the next remainder is exhauſted, by the fourth a third part of the next, and ſo on continually; the quantity of Air evacuated at each ſtroke diminiſhing in the ſame proportion with the quantity of Air remaining in the Receiver immediately before that ſtroke: for 'tis very evident that the third part, or any other determinate part, of any Quantity muſt needs be diminiſhed in the ſame proportion with the whole Quantity it ſelf. And this may ſuffice for the proof of what I aſſerted in the firſt place *viz.* That the Quantities exhauſted at every ſtroke are not equal but are perpetually diminiſhed.

I ſhall now proceed to ſhew, that the Air remaining in the Receiver after every ſtroke is diminiſhed in a Geometrical Progreſſion. It has been prov'd that the Air remaining in the Receiver after each ſtroke of the Pump is to the Air which was in the Receiver immediately before that ſtroke, as the capacity of the Receiver is to the capacity of the Barrel and Receiver taken together, or in other words, that the quantity of Air in the Receiver, by each ſtroke of the Pump, is diminiſh'd in the proportion of the capacity of the Receiver to the capacity of the Barrel and Receiver taken together. Each remainder is therefore evermore leſs than the preceding remainder in the ſame given *Ratio*. That is to ſay theſe remainders are in a Geometrical Progreſſion continually decreaſing. Let us return again to our former Example which may afford a ſomewhat different Light into this matter. The Quantity exhauſted at the firſt turn, you remember, was a third part of the Air in the Receiver, and therefore the remainder will be two thirds of the ſame, and for the like reaſon the remainder after the ſecond turn will be two thirds of the foregoing remainder, and ſo on continually, the decreaſe being always made in the ſame proportion of 2 to 3; conſequently the decreaſing Quantities themſelves are in a Geometrical Progreſſion. It was before proved that the Quantities exhauſted at every turn did decreaſe in the ſame proportion with theſe remainders; therefore the Quantities exhauſted at every turn are alſo in a Geometrical Progreſſion. Let it then be remembred, that the Evacuations and the Remainders do both of them decreaſe in the ſame Geometrical Progreſſion. If the Remainders do decreaſe in a Geometrical Progreſſion, 'tis certain you may, by continuing the Agitations of the Pump, render them as ſmall as you pleaſe, that is to ſay, you may approach as near as you pleaſe to a perfect Vacuum. But notwithſtanding this, you can never entirely take away the Remainder. If it be ſaid that you may, I prove the contrary thus. Before the laſt turn of the Pump, which is ſaid wholly to take away the Remainder, it muſt be confeſs'd there was a Remainder: this Remainder, by that laſt turn of the Pump, will only be diminiſhed in a certain proportion as has been before proved: therefore it was falſly ſaid to be totally taken away.

It may not be improper in this place to ſay ſomething concerning the Gradual Aſcent of the Quickſilver in the Gage, upon which we have made ſome Experiments. You have obſerv'd that as we continue to Pump, the Quickſilver continues to aſcend, approaching always more and more to the Standard Altitude in the Weather-Glaſs, which you know is about

bout $29\frac{1}{2}$ Inches, being a little under or over according to the variety of Seaſons. What I ſhall now endeavour to make out to you is this: that the defect of the height of the Quickſilver in the Gage from the ſtandard Altitude, is always proportionable to the Quantity of Air, which remains in the Receiver: that the Altitude it ſelf of the Quickſilver in the Gage, is proportionable to the Quantity of Air which has been exhauſted from the Receiver: that the Aſcent of the Quickſilver upon every turn of the Pump, is proportionable to the Quantity evacuated by each turn.

In order to underſtand theſe Aſſertions, you are to conſider, that the whole preſſure of the Atmoſphere upon the Ciſtern of the Gage, is equivalent to and may be balanc'd by a Column of Quickſilver of the Standard Altitude. Therefore when in the Gage, the Quickſilver has not yet arrived to the Standard Altitude, 'tis certain the defect of Quickſilver is ſupplyed by ſome other equal force, and that force is the Elaſtick Power of the Air yet remaining in the Receiver, which communicating (as you remember) with the upper part of the Gage, hinders the Quickſilver from aſcending, as it would otherwiſe do, to the Standard Altitude. The Elaſticity of the Air in the Receiver is then equivalent to the weight of the deficient Quickſilver: but the weight of that deficient Quickſilver is proportionable to the Space it ſhould poſſeſs, or to the defect of the height of the Quickſilver in the Gage from the Standard height: therefore the Elaſticity of the remaining Air is alſo proportionable to the ſame defect. And ſince it was formerly proved, that the Denſity of any portion of Air is always proportionable to its Elaſticity, and the Quantity in this Caſe is proportionable to the Denſity; it follows, that the Quantity of Air remaining in the Receiver, is proportionable to the defect of the Quickſilver in the Gage from its Standard Altitude, which was the firſt thing to be proved. Hence it follows, that the Quantity of Air which was at firſt in the Receiver before you began to Pump, is proportionable to the whole Standard Altitude, and conſequently the difference of this Air which was at firſt in the Receiver and that which remains after any certain Number of turns, that is, the Quantity of Air exhauſted, is proportionable to the difference of the Standard Altitude and the beforemention'd defect, that is, to the Altitude of the Quickſilver in the Gage after that Number of turns; which was the ſecond thing to be proved. And from hence it follows that the Quanty of Air exhauſted at every turn of the Pump, is proportionable to the Aſcent of the Quickſilver upon each turn, which was the laſt thing to be made out. And theſe Concluſions do very well agree with the Experiments, which ſhew'd us the quantity of Air that was exhauſted by the Quantity of Water which afterwards ſupply'd the vacant Place of that Air in our Receiver. Let it then be remembred, that the Quantity exhauſted at each turn is proportionable

to

the Aſcent of the Quickſilver upon that turn: that the whole Quantity exhauſted from the time you began to pump is proportionable to the whole Altitude of the Quickſilver: that the quantity remaining in the Receiver is proportionable to the defect of that Altitude from the Standard; to come now to the application of the other Experiments which we made this Day: We found you remember, that the ſeveral Aſcents of the Quickſilver in the Gage, upon every turn of the Pump, were diminiſhed in a Geometrical Progreſſion, and it has juſt now been proved that the quantities of Air exhauſted at each turn are proportionable to thoſe Aſcents. Therefore we may ſafely conclude from Experiment alſo, what we before collected by a train of reaſoning: that the Quantities of Air exhauſted at every turn of the Pump are diminiſhed continually in a Geometrical Progreſſion. Furthermore, ſince thoſe Aſcents are the differences of the defects from the Standard Altitude, upon every ſucceſſive turn of the Pump: it follows, that the defects alſo are in the ſame decreaſing Geometrical Progreſſion. For 'tis a general Theorem, that all Quantities, whoſe differences are in a Geometrical Progreſſion, (ſo long as the Quantities continue to have any magnitude,) are themſelves alſo in the ſame Geometrical Progreſſion. The defects being then in a decreaſing Geometrical Progreſſion, and the Quantities of Air remaining in the Receiver being proportionable (as was lately proved) to the defects: it follows from the ſame Experiments, that the Quantities of Air which remain in the Receiver after every turn of the Pump, do decreaſe in a Geometrical Progreſſion: which was the other thing concluded alſo by a train of reaſoning.

Before I diſmiſs the conſideration of the Air-Pump, it remains that I add ſomething concerning the uſe of the two Tables, which I have put into your hands. They are deſigned to ſhew the number of turns of the Pump, which are requiſite to rarefie, in any given proportion, the Air contain'd under any Receiver. The firſt Table in particular is fitted for Receivers whoſe capacity is the ſame with the capacity of the Barrel, and the Numbers of the firſt Column of it expreſs the degrees of Rarefaction as thoſe over-againſt them in the ſecond Column expreſs the Number of Turns, with their Decimal Parts, which are requiſite to produce thoſe degrees of Rarefaction. Thus for Example, if it were required to rarefie the Air, under ſuch a Receiver, an hundred times above its natural Rarity: I ſeek for the Number 100 in the firſt Column, and over againſt it in the ſecond I find the Number 6,644 by which I underſtand that the Air will be rarified an Hundred times by 6 turns of the Pump and 644 Thouſand parts of a turn. So if it were deſired to rarefie the Air, under the ſame or an equal Receiver, 10 Thouſand times more than in its natural ſtate: I perceive there will be 13 turns and 288 Thouſandth parts of a turn requiſite for that purpoſe. The Receivers which we ſhall have occaſion to make uſe of in our Experiments are generally much bigger than the capacity of each Barrel of the Pump, and by being bigger, will require a greater number of turns than thoſe ſet down in the ſecond Column, to rarifie the Air in the degrees which are expreſs'd in the firſt Column. It may perhaps at the firſt view, ſeem not unreaſonable to think that the number of turns requiſite to rarefie the Air in any certain Degree, ſhould exceed the Numbers of the ſecond Column in the ſame proportion by which the capacity of the Receiver exceeds the capacity of the Barrel. But if the Matter be examined more cloſely, it will be found: that the Number of turns do not increaſe in ſo great a proportion as the capacity of the Receiver does. What that proportion is, by which the Number of turns is truly increaſed, as the capacity of the Receiver becomes bigger, may be ſeen by the ſecond Table. Whoſe firſt Column expreſſes the proportion of the Receiver to the Barrel as the ſecond does the proportion of the true Number of turns to thoſe ſet down in the firſt Table. The uſe of it will be more clearly underſtood by an Example or two. Let us ſuppoſe the capacity of the Receiver to be 10 times greater than the capacity of the Barrel and that we would find how many turns are requiſite to rarefie the Air under ſuch a Receiver 100 times more than it is naturally rarefied. By the firſt Table we find (as was ſaid above) that if the Receiver were equal to the Barrel the Number of turns would be 6,644. But the Receiver is 10 times greater. Find therefore the Number 10 in the firſt Column of the ſecond Table, and over againſt it you will ſee the Number 7,273 in the ſecond Column of the ſame Table, by which you perceive that as the Receiver is increaſed in a Decuple proportion, the Number of turns are increaſed not ſo much, but only in a ſomewhat more than Septuple proportion. Therefore the true Number of turns will be found by multiplying the Number 6,644 by the Number 7,273 and will conſequently be 48, 322. So if it were deſired to find the Number of turns of the Pump, which muſt be made, to rarefie the Air 10 Thouſand times above its natural State, in a Receiver which is 50 times bigger than the capacity of the Barrel: over againſt 10000 in the firſt Table I find 13,288 and over againſt 50, in the ſecond Table, I find 35,003 which multiplyed together make 465,12; This therefore is the Number of turns requiſite for the purpoſe. You need not be ſolicitous about the Fractions which are above any certain whole Number of turns. They do not mean, that the Handle of the Pump is to be moved juſtly ſuch a part of a turn as they ſeem to denote; for ſtricktly ſpeaking it need not be moved altogether, ſo much. But the difference is inconſiderable, and it would be a loſs of time to inſiſt more particularly about it. It was neceſſary to ſet down the Fractions in the Tables that no whole Number of turns might be loſt in the Product, when you come to multiply 'em together; but when you have found the Product, the Fractions belonging to it need not be conſidered.

In making theſe Tables, that they might not be too large, I have as you ſee omitted ſeveral intermediate Numbers. However, they are ſufficient for the purpoſe for which I deſign'd 'em; which was to give you clearer Notions of the Operation of our Engin. I ſhould here explain to you the grounds upon which they were computed; but I fear the difficulty of the Subject would not permit me to be generally underſtood. I ſhall therefore omit the doing of it, and only obſerve to you of the firſt Table, that if you take any Numbers in the firſt Column which are in a Geometrical Progreſſion, the correſpondent Numbers of the ſecond Column will be in Arithmetical Progreſſion. It may alſo be obſerved of the ſecond Table that the diſproportion of the correſpondent Numbers does continually increaſe from the beginning to the end, how far ſo ever it be continued, but yet does never exceed the diſproportion of 13 to 9.

'Tis time now that we proceed to the Condenſer. This Inſtrument will not require much to be ſaid concerning it. When I aſſert that equal Quantities of Air, namely, as much as the Barrel can naturally contain, are intruded into the Receiver at each ſtroke of the Forcer: the thing is ſo very obvious that I believe I need not go about to prove it. For you cannot but eaſily underſtand, that as the Embolus or Forcer is drawn upwards from the bottom of the Barrel, there is a vacuity left behind it, till ſuch time as it comes to get above the little Hole which is made in the ſide of the Barrel towards the top of it. For then the external Air is permitted to paſs freely through that Hole into the aforeſaid void Space, and conſequently the Barrel will then have as much Air in it as it can naturally contain. And as the Forcer is moved downwards this Air is compreſs'd, and by compreſſion is more and more condens'd till at length the force of its Elaſticity becomes greater than the Elaſtick force of that which is contain'd within the Receiver, and thereby it will open the Valve and make way for it's ſelf to enter totally into the Receiver as it is continually puſh'd forwards by the deſcending Embolus. Since then the Quantities intruded at each ſtroke of the Forcer are equal, it manifeſtly appears that the Quantities in the Receiver and conſequently the degrees of Condenſation do increaſe in an Arithmetical Progreſſion. Let us now examine by what ſteps the Quickſilver in the Gage advances at each ſtroke. What I ſhall endeavour to prove as to this matter is this: that as the Quickſilver is moved forwards in the Gage upon every ſucceſſive ſtroke of the Forcer, the Spaces at the end of the Gage, which are yet left free from the Quickſilver, do decreaſe in a Muſical Progreſſion. But in the firſt place it may not be amiſs to explain in ſome meaſure the nature of Muſical Progreſſions, ſince theſe are not generally ſo well underſtood as thoſe which we call Arithmetical and Geometrical Progreſſions. In order to do this, I ſhall propoſe an Inſtance which firſt gave occaſion for the Name. 'Tis a thing well known among Muſicians, if three Chords

or

or Strings, in all other reſpects alike, be of different lengths, and thoſe lengths be to each other in proportion as the Numbers 6, 4 and 3, that the Sounds of thoſe Strings will expreſs the principal and moſt perfect of the Muſical Concords, namely, and Eight a Fifth and a Fourth. Thus the Sound of the laſt will be an Octave to the Sound of the firſt, and the Sound of the ſecond a Fifth to the Sound of the firſt, and the Sound of the laſt a Fourth to the Sound of the ſecond. Hence theſe Numbers 6, 4 and 3, which expreſs the Proportions of thoſe Muſical Strings were ſaid not improperly to be in a Muſical Progreſſion. Now it was eaſy to be obſerved that theſe Numbers were reciprocally proportionable to three other Numbers reſpectively, *viz.* 2, 3 and 4 which were in Arithmetical Progreſſion; and thence it came to paſs, that any other Series of Numbers was ſaid to be in a Muſical Progreſſion which had the ſame property of being reciprocally proportionable to a Series of Numbers in Arithmetical Progreſſion. That therefore is a Series of Muſical proportionals which is reciprocal to another Series of Arithmetical proportionals. But beſides this, you may obſerve another property belonging to the above mention'd Numbers 6, 4 and 3 *viz.* That the Firſt is to the Third as the difference of the Firſt and Second is to the difference of the Second and Third. And this property does equally belong to all other Numbers, which are reciprocally as a Series in Arithmetical Progreſſion, that is, to all other Numbers which are in a Muſical Progreſſion. Hence if any two ſucceeding Terms be given, the Third may be found by dividing the Product of the Firſt and Second by the difference which ariſes in Subſtracting the Second from the double of the Firſt. Thus in the Progreſſion 6, 4, and 3 the Product of the firſt and ſecond Terms 6 and 4 is 24, and the difference which ariſes by ſubſtracting the ſecond Term (4) from the double of the firſt (12) is 8, and the Quotient which emerges by dividing the Product (24) by the difference (8) is 3 the third Term in the Progreſſion required. I ſhall now go on to ſhew that the Spaces unpoſſeſs'd by the Quickſilver at the end of the Gage do decreaſe in ſuch a Muſical Progreſſion. It muſt be obſerved therefore, that the Quickſilver of the Gage is contiguous on one ſide to the Air within the Receiver and on the other ſide to the Air which is ſhut up at the end of the Gage, and the Denſity of the Air in both Places is equal. For were the Denſity of the Air in the Receiver greater than the Denſity of the Air at the end of the Gage; its Elaſtick Force would alſo be greater, and by that Exceſs of Force the Quickſilver would be moved on further towards the end of the Gage, till the Forces and conſequently the Denſities became equal. After the ſame manner if the Denſity of the Air at the end of the Gage were greater than the Denſity of the Air within the Receiver; the Quickſilver would be moved backwards from the end of the Gage, till the Denſities became equal. It is manifeſt therefore that the Denſities are equal on both parts when the Quickſilver in the Gage is at reſt. Therefore ſince the Denſity of the Air in the Receiver upon every ſucceſſive ſtroke of the Forcer was increaſed in Arithmetical Progreſſion, it follows that the Denſity of the Air at the end of the Gage is likewiſe increaſed in the ſame Arithmetical Progreſſion. But the Space which that Air poſſeſſes is diminiſhed in the ſame proportion by which the Denſity is increaſed, or in other words, the Spaces are reciprocally as the Denſities: therefore the Spaces are reciprocally as a Series of Terms in Arithmetical Progreſſion, which is the ſame thing as to ſay, the Spaces are in a Muſical Progreſſion. And this concluſion we found alſo to agree with our Experiments.

A Second Enquiry into the State of the Atmoſphere.

BEFORE I conclude it may not be amiſs in this Place to make our Enquiries once more concerning the State of the Atmoſphere, and the different degrees by which the Air is rarified at different Altitudes above the Surface of the Earth. You remember it was proved in the foregoing Week, that the Denſity of the Air was diminiſhed in a Geometrical Progreſſion as the Altitude of it was increaſed in an Arithmetical Progreſſion. The truth of that Rule depends upon this Suppoſition, that the Gravity of Bodies is the ſame at all diſtances from the Centre of the Earth. But it has been proved and put beyond diſpute by Sir *Iſaac Newton* in his *Principia*, that the Gravity of Bodies is not exactly the ſame at all diſtances from the Centre, but is diminiſhed as the diſtance increaſes, ſo that the Quantity of it is always reciprocally proportionable to the Square of the diſtance. From hence it eaſily appears that when the Altitude of the Air above the Surface of the Earth is very great and very conſiderable in reſpect of the Earth's Semidiameter, the Rule which I formerly gave you will be far from being true; but if the Altitude be ſmall and inconſiderable (as the Altitudes of our higheſt Mountains muſt be confeſs'd to be) it will ſtill be ſufficiently exact, and as ſuch it is propoſed by Dr. *Halley* in the Philoſophical Tranſactions, and by Dr. *Gregory* in his Aſtronomy, and generally received by others without any exceptions. However, it may be worth our while to ſee what conſequences will ariſe upon the truer Hypotheſis which ſuppoſes (as I ſaid above) the Gravity of Bodies to be diminiſhed in the ſame proportion by which the Square of their diſtance from the Centre of the Earth is increaſed. In treating of this matter I fear I ſhall not be generally underſtood, yet I hope I ſhall make the thing as eaſy as the nature of it will permit. (Fig. 15.) Let *C* repreſent the Centre of the Earth, *CA* its Semidiameter, *AB* a part of its Surface, and let the Line *CAD* be produced up to the extremity of the Atmoſphere. In this Line imagine the Points

D, *E*, *F*

D, E, F to be placed infinitely near to each other, and take as many other Points d, e, f in ſuch a manner that the diſtances dC, eC, fC ſhall be reciprocally proportionable to the diſtances DC, EC, FC reſpectively, or in ſuch manner that the diſtances dC, eC, fC ſhall be leſs than the Semidiameter AC in the ſame proportion by which the reſpective diſtances DC, EC, FC are greater than the ſame Semidiameter: the diſtances of the leſſer Letters from the Centre being diminiſhed in the ſame proportion by which the diſtances of the correſponding greater Letters from the Centre are increaſed. Upon the Points A, d, e, f erect the Perpendiculars AB, dp, eq, fr, and ſuppoſe the length of theſe Perpendiculars to be proportionable to the Denſity of the Air in A, D, E, F reſpectively ſo that the Denſity of the Air at A ſhall be repreſented by the Perpendicular AB, the Denſity of the Air at D by the Perpendicular dp, the Denſity at E by the Perpendicular eq, and the denſity at F by fr. This being done, I am now to prove, that if the diſtances CF, CE, CD be taken in a Muſical Progreſſion and conſequently the diſtances Cf, Ce, Cd, be in an Arithmetical Progreſſion as being reciprocally proportionable to the former diſtances; the Perpendiculars fr, eq, dp, and conſequently the Denſities of the Air in the places F, E, D which are analogous to the Perpendiculars will be in a Geometrical Progreſſion. In the firſt place then becauſe the diſtances of the leſſer Letters from the Centre are reciprocally as the diſtances of their correſpondent greater Letters from the ſame, it is manifeſt that Cd is to Ce as CE is to CD, and conſequently the difference of Cd and Ce is to the difference of CE and CD as Ce to CD, or (becauſe the Points E and D are ſuppoſed to be infinitely near to each other) as Ce to CE or (becauſe Ce is leſs than CA in the ſame proportion by which CE is greater than CA, and conſequently Ce, CA and CE are continual proportionals) as CAq is to CEq. It is evident then, that de (the difference of Cd and Ce) is to DE (the difference of CE and CD) as CAq is to CEq. Therefore if the diſtance CE remain unaltered and conſequently the proportion of CAq to CEq remain unaltered, the proportion of de to DE will alſo remain unaltered, and conſequently de will be as DE, that is, de will be increaſed and diminiſhed in the ſame proportion with DE. But if DE remain unaltered, becauſe it is always greater than de in the proportion by which CEq is greater than CAq, it follows that de muſt neceſſarily be diminiſh'd in the ſame proportion by which CEq is increaſed, and increaſed in the ſame proportion, by which CEq is diminiſhed, or in other words it muſt always of neceſſity be reciprocally as CEq; whence it follows, that if neither DE nor CE remain unaltered, de will be as DE directly and as CEq reciprocally. But the Bulk of Air between the places D and E is as DE, and the gravity of the ſame is reciprocally as the Square of CE it's diſtance from the Centre: therefore de is as the Bulk and Gravitation together of the ſame, and conſequently ly ſince *eq* is as its Denſity the Product of *de* and *eq* or the Area *deqp* will be as the Product of its Denſity, Bulk, and Gravitation, that is, as its Force to compreſs the inferior Air. And the ſum of all ſuch Areas below *dp* will be as the ſum of ſuch Forces of all the Air above *D*, that is, as *dp* the Denſity of the Air at *D*, for you know the Denſity of the Air is always as the Force which compreſſes it. Since the Perpendicular *dp* is as the ſum of all the little Areas below its ſelf and the Perpendicular *eq*, for the ſame reaſon, is as the ſum of all below its ſelf: it follows, that the difference of *eq* and *dp* is as the difference of thoſe Sums, which difference is the Area *eqpd*. Thus far then we have proceeded, we have found that the difference of the Perpendiculars *eq* and *dp* is as the Area *eqpd* comprehended by thoſe Perpendiculars. Let us now ſuppoſe the diſtances *CF*, *CE*, *CD*, and ſo on, to be taken in a Muſical Progreſſion and then (as was ſaid above) the diſtances *Cf*, *Ce*, *Cd*, and ſo on, will be in an Arithmetical Progreſſion, and therefore all the Intervals *de*, *ef* will be equal and conſequently the Areas *eqpd* which have thoſe equal Intervals for their Baſes will be as their Altitudes *eq*. Hence the difference of *eq* and *dp* which was as the Area *eqpd* will be as *eq* and conſequently *dp* will be as *eq*. In other words, the two Perpendiculars which terminate the little Area included between them do every where bear the ſame given proportion to each other. Therefore the proportion of *fr* to *eq* is the ſame with the proportion of *eq* to *dp*, and conſequently the Perpendiculars *fr*, *eq*, *dp*, and ſo on, are in a Geometrical Progreſſion. But theſe Perpendiculars expreſs the Denſities of the Air at the places *F*, *E*, *D*, and ſo onwards. Therefore thoſe Denſities are alſo in a Geometrical Progreſſion, which was the thing to be proved. To proceed further: ſince *Cd* is to *CA* as *CA* is to *CD*, it follows that *Ad* is to *AD* as *CA* to *CD*, or in other words that *Ad* is leſs than *AD* in the ſame proportion by which the Semidiameter of the Earth is leſs than the diſtance of the Point *D* from the Centre; conſequently to find the length of *Ad* we muſt diminiſh the Altitude *AD* in the proportion of the Semediameter of the Earth to the ſum of the Semidiameter and the Altitude, for which reaſon I ſhall call *Ad* the diminiſh'd Altitude of the Point *D*, and upon the ſame account *Ae* may be call'd the diminiſh'd Altitude of the Point *E*, and *Af* the diminiſh'd Altitude of the Point *F*; and ſo if *h* be the Point which correſponds as above to the Point *H*, *Ah* will be the diminiſh'd Altitude of the Point *H*. Now 'tis eaſy to obſerve that as the diſtances *Cd*, *Ce*, *Cf* are in Arithmetical Progreſſion, ſo are alſo the diminiſh'd Altitudes *Ad*, *Ae*, *Af*. And from hence there ariſes this Theorem. That if the diminiſh'd Altitudes be taken in Arithmetical Progreſſion the Denſities of the Air will be in a Geometrical Progreſſion. Therefore if the Rarity of the Air at any one Altitude, ſuppoſe at *H*, be known, you may eaſily enough find its Rarity at any other Altitude ſuppoſe at *D*. For as the diminiſh'd Altitude of the Point *H* is to the diminiſh'd Altitude of the Point *D*, ſo will be the Logarithm of the Air's Rarity at *H* which is ſuppoſed to be known, to the Logarithm of the Airs Rarity at *D* which was to be found. The whole difficulty of the buſineſs is therefore reduc'd to this: to find the Rarity of the Air at ſome one Altitude as at *H*. This may be done as I formerly ſhew'd you by carrying the Barometer to the top of ſome very high Mountain and obſerving the deſcent of the Quickſilver. Such were the Experiments made upon the *Puy de Domme* in *France*, and *Snowden Hill* in *Wales*, which I made uſe of the laſt Week when I diſcours'd of this Subject. But the Method I ſhall now deſcribe to you is more expeditious and depends upon the Experiment which we made this Day. It appeares (as I ſaid) by many ſuch Experiments compar'd together, that the weight of Air is to the weight of Water as 1 to about 850. Therefore a Column of Air whoſe height is 850 Inches or 70 Feet and 10 Inches will be equal in weight to a Column of Water upon the ſame Baſis whoſe height is 1 Inch. Let us ſuppoſe that *AH* the height of the Point *H* above the Surface of the Earth is 70 Feet and 10 Inches; then becauſe the Standard height of Water in the Paſcalian Tube is 34 Feet or 408 Inches, and this height of Water is a Ballance to the Preſſure of the whole Atmoſphere upon the Surface of the Earth, 'tis manifeſt that the weight of the whole Column of Air, which is ſuperiour to the Point *A*, is equal to the weight of a Column of Water upon the ſame Baſis whoſe height is 34 Feet or 408 Inches. Take from the weight of the whole Column of Air the weight of that part of the Column which reaches from *A* up to *H*, and which was ſhewn to be equal to one Inch of Water; and the weight of the remaining part of the Column which is above the the Point *H* will be equal to the weight of 407 Inches of Water. Therefore the Force with which the Air at *A* is compreſs'd is to the Force with which the Air at *H* is compreſs'd as 408 to 407, and the Rarity of the Air at *H* is to the Rarity of the Air at *A* in the ſame proportion. You may perceive that this Method ſuppoſes the Air to be of the ſame Denſity in every part of the Space *AH*, which is not exactly true; but in ſo ſmall an Altitude as that of 70 Feet the Error is altogether inſenſible. However if you have a mind to proceed with the utmoſt accuracy, you may do ſo, by making the Altitude as ſmall as you pleaſe.